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# STRENGTH — STRESS MODEL OF THE WEAR-OUT PROCESS Part two

## ABSTRACT

In the previous part (i.e. part one) of this paper [1] a differential equation intended to model a wide class of physical failure processes that may take place in technical devices was put forward. What relates model's applicability — among the multitude of users of technical devices there surely will be ones that find the model to be relevant to real processes that cause their devices to fail. A set of relevant devices comprises these ones that are exploited (in an exact meaning of this word). Let us remember a term strength that is crucial to the model. The failure results from loss of strength that, in turn, is impacted by external stress.

In part one we 'injected' randomness into physics expressed by means of the differential equation. Parameters that govern failure process in question were treated as random variables. Each failure process ends in failure, so time to failure is the random variable. In part two of this paper we will, with the Monte Carlo method, investigate how variability of process parameters shapes the hazard rate function.

Key words:
strength, stress, load, wear-out phenomenon, random variable, Monte Carlo method, hazard rate function, failure pattern.

## THE FAILURE PATTERN

Since the reliability originated as a branch of engineering, the reliability pattern named 'bathtub hazard rate' has been coined. The pattern is presented in figure 1. It covers the whole device's lifetime comprising three lifetime intervals (li).

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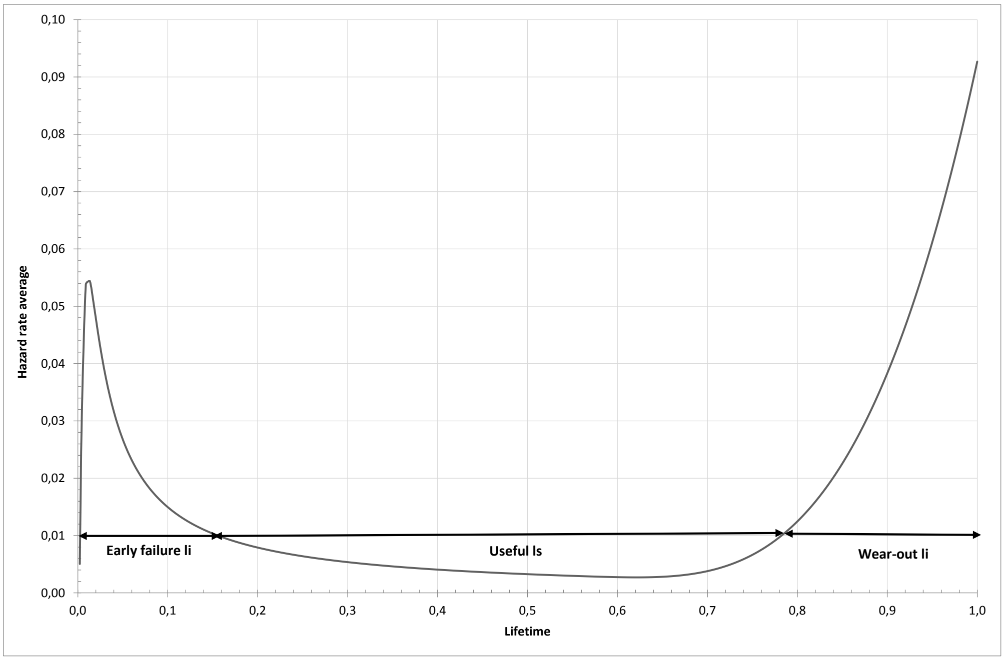


Fig. 1. The failure pattern

It was [2] and still is [3] a common belief that all the failures, regardless, in what device they occur, follow the bathtub pattern. The reader would do well to notice that the bathtub failure pattern is attributed to batches of devices that are heterogeneous in sense of causes of failures. It will be shown in next section, that the bathtub may also 'produce' the hazard rate function that fits the failure pattern in question.

## MONTE CARLO EXPERIMENTS

There are three variables in the model which we may assume be random, namely $p, S_o, v_o$. If so, the following question is sure to arise immediately: of what forms distributions of $p, S_o$ and $v_o$ are? Doubtless, exact distributions of these values remain unknown forever. We can only arbitrarily, but reasonably, choose one from distributions commonly accepted in reliability mathematics. These are the normal, lognormal and, of course, Weibull ones. Choosing distribution is a risky matter but not such risky one as setting concrete parameter values is. The author is not a risk-taker to provoke choice-related discussion. Therefore, we assume $p, S_o, v_o$ be uniformly distributed. The uniform distribution can be attributed as 'worst case' (any value is equally possible) distribution when compared to these mentioned above that are more or less concentrated around their expected values.

In this paper results of three numerical experiments carried out with the model are reported. Each numerical experiment was composed of eight sub-experiments marked by capital letters from A to H. In each experiment only one parameter was randomized. Other parameters kept their particular constant values in each sub-experiment. Values of parameters of the model are presented in tables numbered from 1 to 3. Rows of the tables are attached to model parameters. Some parameters namely $v_o, p, S_o$ have two rows attached. These contain lower and upper bounds of the interval in which particular parameter are uniformly distributed. Columns, except the leftmost one, are attached to sub-experiments. Each sub-experiment was performed in two steps:

Step 1: 1000 random lifetimes $t_f$ were generated according to (6a) of the first part.

Step 2: Lifetimes were ordered ascendingly, then the empirical reliability function and the hazard rate average were estimated (see Appendix).

**Numerical experiment No. 1**: The randomized parameter: initial strength $S_o$.

Intervals of variability are of the same length but relative variability decreases as the lower bound is higher and higher. Population of devices in H sub-experiment is much more homogeneous with respect to strength than in A is. It is worth of notice how the hazard rate evolves from strictly increasing to bathtub one.

Table 1. Parameter values related to experiment No. 1

| | Index of parameter combination | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **A** | **B** | **C** | **D** | **E** | **F** | **G** | **H** |
| **$v_o$** | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 |
| **p** | 5,00 | 5,00 | 5,00 | 5,00 | 5,00 | 5,00 | 5,00 | 5,00 |
| **$S_o$** | **0,50** | **1,00** | **1,50** | **2,00** | **2,50** | **3,00** | **3,50** | **4,00** |
| | **1,00** | **1,50** | **2,00** | **2,50** | **3,00** | **3,50** | **4,00** | **4,50** |
| **$S_{cr}$** | 0,40 | 0,40 | 0,40 | 0,40 | 0,40 | 0,40 | 0,40 | 0,40 |
| **$L_o$** | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 |

**Numerical experiment No. 2:** The randomized parameter: degradation speed $v_o$.

This experiment was intended to study a way the degradation speed $v_o$ shapes the hazard rate function. Variability of $v_o$ increases and in experiment labeled H is thirteen times greater than in labeled A! It causes the hazard rate function to evolve from strictly increasing to bathtub one.

Table 2. Parameter values related to experiment No. 2

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| **$v_o$** | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 |
| | 0,02 | 0,03 | 0,04 | 0,05 | 0,06 | 0,07 | 0,10 | 0,13 |
| **p** | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 |
| **$S_o$** | 2,00 | 2,00 | 2,00 | 2,00 | 2,00 | 2,00 | 2,00 | 2,00 |
| **$S_{cr}$** | 0,40 | 0,40 | 0,40 | 0,40 | 0,40 | 0,40 | 0,40 | 0,40 |
| **$L_o$** | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 | 0,10 |

**Numerical experiment No. 3:** The randomized parameter: initial strength $S_o$.

In this experiment variability of $S_o$ is equal in all sub-experiments. The interval in which $S_o$ varies is very wide. Upper bound is four times greater than lower one! A phenomenon we observe in figure is all governed by the $p$ parameter. To understand an essence of this experiment let us look at formula (1). The greater $p$ the higher speed devices are losing their strength. When $p$ increases, the hazard rate evolves from strictly increasing to bathtub one. It is because devices become more sensitive to load imposed on them.

Table 3. Parameter values related to experiment No. 3

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| **$v_o$** | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 | 0,01 |
| p | 0,5 | 1,0 | 1,5 | 2,0 | 2,5 | 3,0 | 3,5 | 4,0 |
| **$S_o$** | 0,5 | 0,5 | 0,5 | 0,5 | 0,5 | 0,5 | 0,5 | 0,5 |
| | 2,0 | 2,0 | 2,0 | 2,0 | 2,0 | 2,0 | 2,0 | 2,0 |
| **$S_{cr}$** | 0,4 | 0,4 | 0,4 | 0,4 | 0,4 | 0,4 | 0,4 | 0,4 |
| **$L_o$** | 0,45 | 0,45 | 0,45 | 0,45 | 0,45 | 0,45 | 0,45 | 0,45 |

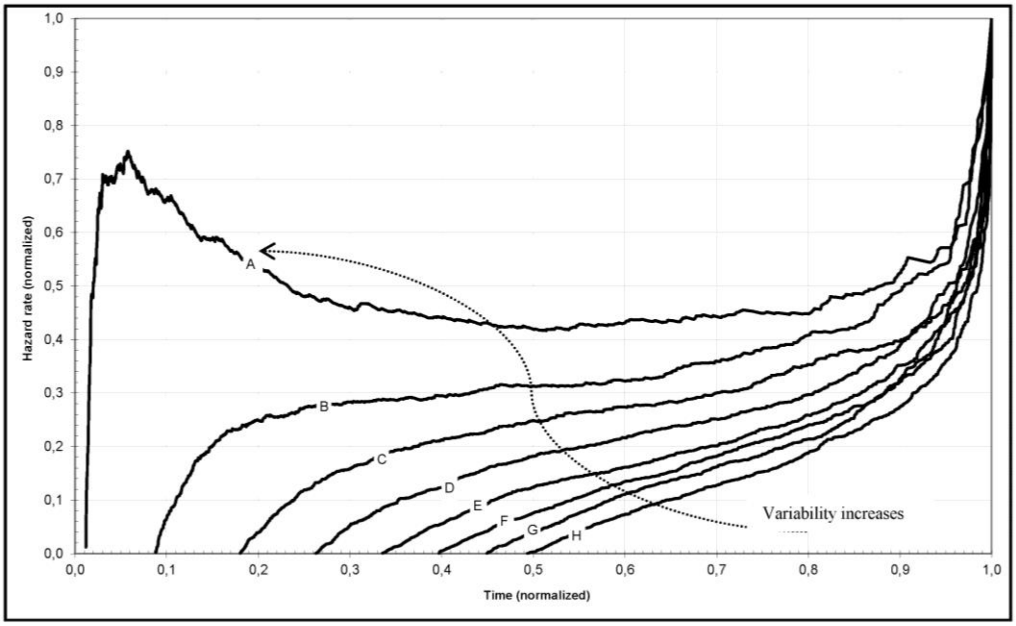


Fig. 2. The hazard rate functions resulting from experiment No. 1

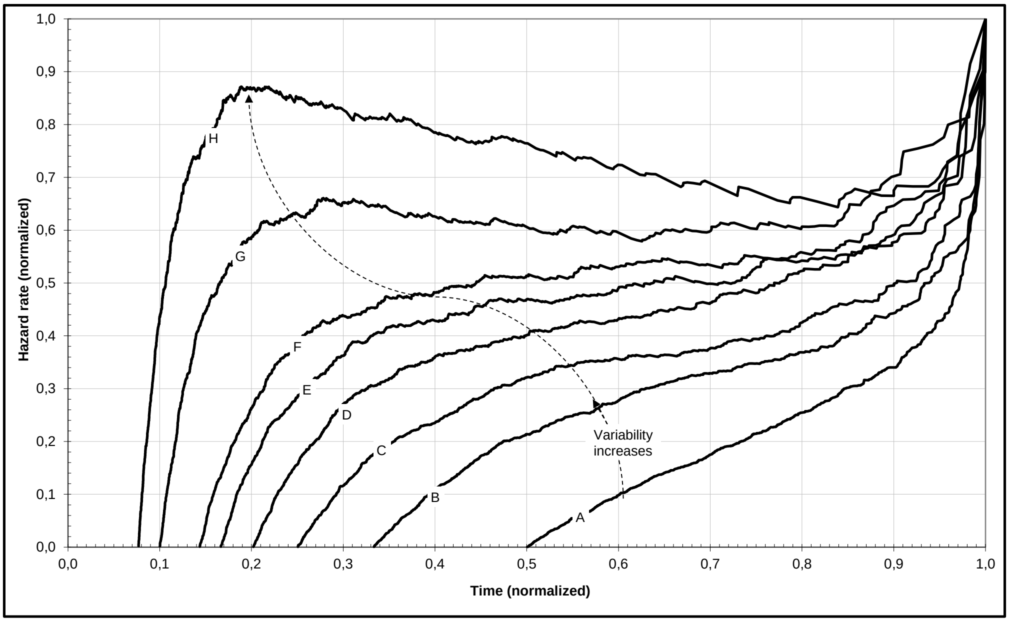


Fig. 3. The hazard rate functions resulting from experiment No. 2

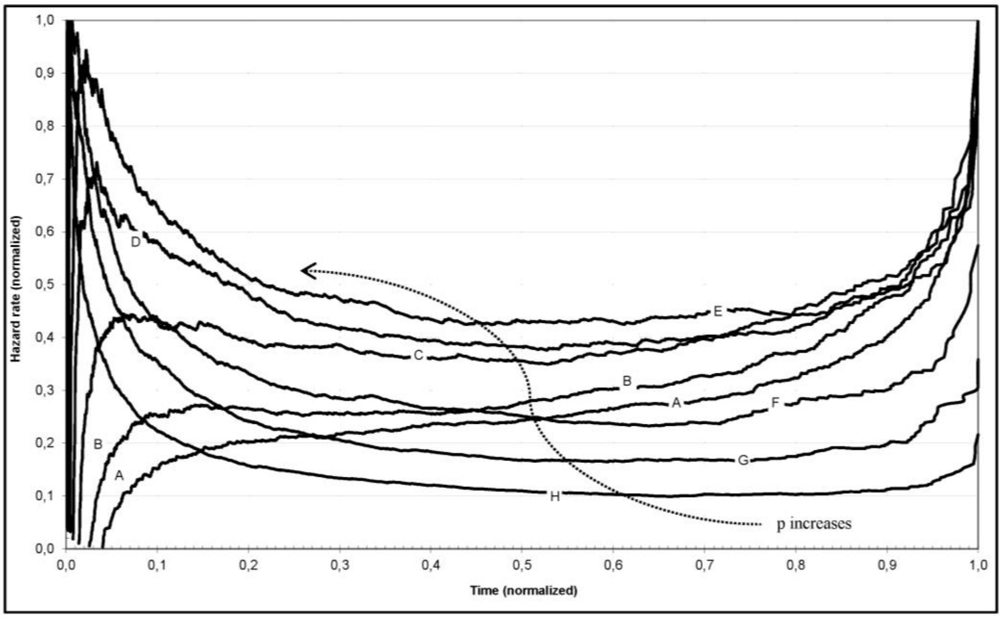


Fig. 4. The hazard rate functions resulting from experiment No. 3

## CONCLUSION

We obtained a rich collection of hazard rate functions: From strictly increasing to these of bathtub shape. All of them have been obtained with the same failure processes! So far that the bathtub failure pattern was attributed to batches of devices that are heterogeneous in sense of causes of failures. It was shown in this paper, that one and homogeneous failure process may also 'produce' the hazard rate function that fits the failure pattern in question. It takes place when a set of individual trajectories is varied. If not, the corresponding hazard rate function is strictly increasing. Let the following slogan end the paper: Early failures are also wear-out failures.

## APPENDIX

Let us remember the functions that characterize reliability. There are four characteristics namely.

The reliability function $R(t)$, failure density function $f(t)$ and hazard rate function $h(t)$. These characteristics are interrelated:

the reliability function also called the survival function

$$R(t) = \Pr\left(t_f \geq t\right), \tag{1a}$$

where $t_f$ is time to failure;

the failure density function

$$f(t) = -\frac{d}{dt} R(t)\,; \tag{1b}$$

the hazard rate function

$$h(t) = f(t)/R(t)\,; \tag{1c}$$

the reliability function expressed by means of the hazard rate function

$$R(t) = \exp\left[-\int_0^t h(u)du\right]; \tag{1d}$$

the hazard rate average $\bar{h}(t)$:

$$\bar{h}(t) = \frac{1}{t} \cdot \int_0^t h(u)du \overset{(8d)}{=} -\frac{1}{t} \cdot \ln\left[R(t)\right]. \tag{1e}$$

It is a very useful substitute of the hazard rate that does not need probability density function $f(t)$ to be estimated. Let $t_1^*, t_2^*, \ldots, t_i^*, \ldots, t_n^*$ be lifetimes observed on sample of $n$ items and $t_{(1)}^*, t_{(2)}^*, \ldots, t_{(i)}^*, \ldots, t_{(n)}^*$ be the same lifetimes but ordered increasingly (of course, in general $t_i^* \neq t_{(i)}^*$). The hazard rate average estimator has a form:

$$\bar{h}_{(i)}^* = \bar{h}\left(t_{(i)}^*\right) = \frac{1}{t_{(i)}^*} \cdot \ln\left(1 - \frac{i}{n+1}\right); \; i = 1, 2, \ldots, n\,. \tag{2}$$

## REFERENCES


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# MODEL PROCESU ZUŻYCIA TYPU WYTRZYMAŁOŚĆ — OBCIĄŻENIE Część druga

## STRESZCZENIE


W drugiej części artykułu przedstawiono schemat eksperymentu numerycznego przeprowadzonego metodą Monte Carlo. Eksperyment polegał na generowaniu czasów pracy do uszkodzenia obiektów, w których zachodzi proces zużycia typu wytrzymałość — obciążenie, opisany w części pierwszej [1]. Zrealizowano eksperyment trzykrotnie, oznaczając realizacje jako No. 1, No. 2 i No. 3. W każdej realizacji inny z parametrów mających wpływ na przebieg procesu uszkodzeniowego czyniono zmienną losową. Realizacje eksperymentu przyniosły bogaty materiał statystyczny, bowiem w każdej z nich przebieg procesu powtarzano tysiąc razy. Wyznaczono doświadczalnie uśrednione funkcje ryzyka uszkodzenia. Porównano je z tak zwanym wzorcem uszkodzeniowym, czyli funkcją ryzyka o przebiegu wannowym, która jest powszechnie tłumaczona w literaturze niezawodnościowej różnorodnością procesów uszkodzeniowych zachodzących w obiektach technicznych tworzących tę samą populację generalną. W artykule poddano w wątpliwość to tłumaczenie, pokazując, że wannowa funkcja ryzyka może być obserwowana w populacjach obiektów technicznych, w których zachodzi tylko jeden proces prowadzący do uszkodzenia. Pokazano płynne przejście od funkcji ryzyka monotonicznie rosnącej do wannowej powodowane jedynie zwiększeniem wariancji losowego parametru wpływającego na przebieg procesu uszkodzeniowego.



Słowa kluczowe:

wytrzymałość, obciążenie, proces zużycia, proces uszkodzeniowy, zmienna losowa, metoda Monte Carlo, funkcja ryzyka uszkodzenia, wzorzec uszkodzeniowy.