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MATERIAL ORTHOTROPY IN ANALYSIS OF FATIGUE SAFETY UNDER STOCHASTIC LOADS

ABSTRACT

The paper deals with fatigue safety of structural elements made of orthotropic metallic materials and subjected to non-zero mean stochastic loads. Multiaxial stationary stress is considered under assumption that the power spectral densities of its components at a given point of the element are known. A reduced stress, equivalent to the original stress in terms of fatigue performance of the material, is determined. For this purpose the distortion energy strength theory and the theory of energy transformation systems are applied. Hereby physical parameters of the material associated with the stress components acting on the plane perpendicular to the orthotropy axis and those associated with the remaining stress components are taken into account. As a result, the formula for expected value of the fatigue safety factor in such a case is derived.

Keywords:
fatigue safety, stochastic loads, orthotropic materials.

INTRODUCTION

Most metals are composed of many microscopic crystals, or grains, each of which has different properties in different directions and all of which are oriented more or less at random. However, from an engineering point of view we are not concerned with the anisotropy of single crystals as such, because single crystals are not generally used in structural applications, but we are concerned with the anisotropy of single crystals insofar as it causes anisotropy of polycrystalline engineering materials in which the fabrication process has caused all the individual crystals (or grains) to be oriented similarly. This condition of non-random orientation of the grains is referred to as 'preferred orientation' [7] which enables the anisotropy to be taken into account in engineering calculations. In particular, even small amounts of longitudinal prestrain can cause a considerable difference in the longitudinal and transverse yield stress, whereas cold rolling of a wide plate results in a lengthening and thinning so that the anisotropy exhibits three mutually perpendicular planes of symmetry [7].

When the anisotropy possesses an axis of symmetry, it is said to be 'orthotropic symmetry'. In the present paper a fatigue safety of orthotropic metallic materials under multiaxial stochastic loads with non-zero mean values is considered. For this purpose the Cartesian reference system *Oxyz* is introduced in such a way that its *z*-axis is in common with the orthotropy axis of the material. The stress components are denoted σ_i where $i = x,y,z,xy,zx,zy$. We assume that the physical parameters of the material associated with the stress components σ_z, σ_{zx} and σ_{zy} acting on the plane perpendicular to the *z*-axis, i.e. Young modulus *E*, tensile yield stress R_e, shear yield stress R_{es}, Poisson's ratio υ and fatigue limits Z_{rc}, Z_{go} and Z_{so} under symmetric tension-compression, bending and torsion, are given. We also assume that the analogous parameters $E', R'_e, R'_{es}, \upsilon', Z'_{rc}, Z'_{go}, Z'_{so}$ associated with the stress components σ_x, σ_y and σ_{xy} are known. For the sake of brevity, the stress components σ_y and σ_{zy} will be dropped.

Fatigue design criteria vary from these for fatigue 'safe life', i.e. for structural elements loaded below the fatigue limit, to those for limited fatigue life, where a certain degree of fatigue damage accumulated in the material during the service life may be accepted [4, 5]. The subject of this paper is the fatigue safety of structural elements fabricated from orthotropic materials and designed for the fatigue 'safe life' regime.

FATIGUE SAFETY IN A REDUCED STRESS STATE

In the general case of multiaxial stochastic loads, the Cartesian stress components at a given point of the element are determined as

$$\sigma_i(t) = \bar{\sigma}_i + \tilde{\sigma}_i(t), \tag{1}$$

where:

$\bar{\sigma}_i$ — mean value of *i*-th stress component;

$\tilde{\sigma}_i$ — *i*-th zero mean stochastic process.

It is assumed that $\bar{\sigma}_i$ are deterministic quantities, and that $\tilde{\sigma}_i$ are stationary (in the wide sense) and stationary correlated processes of known power spectral densities and cross power spectral densities.

In fatigue analysis of structural elements subjected to multiaxial stresses, usually an attempt is made to calculate a reduced stress equivalent in terms of fatigue performance to the original stress, and to deal with the uniaxial stress model. Hereunder the reduced stress will be expressed in the form of a periodic (in the mean-square sense) normal stress in the direction of z-axis as a Gaussian process [5]

$$\begin{aligned}\sigma_e(t) &= \overline{\sigma}_e + a\sin(\omega_e t + \varphi) = \\ &= \overline{\sigma}_e + a_1 \exp(j\omega_e t) + a_{-1}\exp(-j\omega_e t)\end{aligned}, \tag{2}$$

where:

a — random amplitude;
j — imaginary unity;
$\overline{\sigma}_e$ — mean value (deterministic quantity);
φ — random phase angle;
ω_e — circular frequency (deterministic quantity),

and

$$\begin{aligned}&a_1 = \frac{a}{2j}\exp(j\varphi), \quad a_{-1} = a_1^* \\ &\hat{E}\{a_1\} = \hat{E}\{a_{-1}\} = \hat{E}\{a_1^* a_{-1}\} = \hat{E}\{a_{-1}^* a_1\} = 0\end{aligned}. \tag{3}$$

Here $\hat{E}\{\cdot\}$ denotes the expected value and $(\cdot)^*$ the complex conjugate.

In order to evaluate the fatigue safety of structural elements subjected to uniaxial normal stress below the fatigue limit, it is convenient to define a fatigue safety factor. As such, in [5, 6] the following quantity is used for asymmetric tension-compression

$$f = f_d\left(1 - f_s^{-1}\right), \tag{4}$$

where

$$f_d = \frac{Z_{rc}}{\sigma_a} \quad , \quad f_s = \frac{R_e}{\sigma_m} \tag{5}$$

are the partial safety factors related to amplitude σ_a and mean value σ_m of the original stress.

Consequently, the expected value of the fatigue safety factor will be given by

$$\hat{E}\{f\} = \hat{E}\{f_d\}\left(1 - f_s^{-1}\right) = \frac{Z_{rc}}{\hat{E}\{a\}}\left(1 - \frac{\overline{\sigma}_e}{R_e}\right). \tag{6}$$

So, the criterion of fatigue 'safe life' under the reduced stress (2) reads

$$\hat{E}\{f\} > 1 \tag{7}$$

i.e.

$$\frac{\hat{E}\{a\}}{Z_{rc}}+\frac{\overline{\sigma}_e}{R_e}<1\,. \tag{8}$$

In what follows the expected value of the amplitude and mean value of the reduced stress will be sught with material orthotropy taken into account.

REDUCED STRESS

For the determination of reduced stress equivalent to the multiaxial stress with components (1), a multiaxial fatigue strength theory must be applied. In the case of multiaxial stress with constant principal stress directions, the Huber-Mises--Hencky (H-M-H) distortion energy theory is considered to be satisfactory [8]. However, if additionally the theory of energy transformation systems [2] with dissipative energy as a scalar parameter is utilized, then the requirement of invariance of the principal stress system can be avoided [5]. Since the spectral methods lead to substantial savings of calculation time [10], the H-M-H theory and the theory of energy transformation systems will be here implemented to time-varying stresses in the frequency domain.

In the case of isotropic materials adaptation of the H-M-H theory to a multiaxial stress with components $\sigma_x, \sigma_z, \sigma_{xy}$ and σ_{zx} leads to equation [1, 3]

$$\frac{1+\upsilon}{3E}\sigma_e^2=\frac{1+\upsilon}{3E}\left[\sigma_x^2+\sigma_z^2-\sigma_x\sigma_z+3\left(\sigma_{xy}^2+\sigma_{zx}^2\right)\right]. \tag{9}$$

To account for the orthotropy of the material, we introduce the Young modulus E' and Poisson's ratio υ' associated with the stress components σ_x and σ_{xy} by writing

$$\begin{aligned}\frac{1+\upsilon}{3E}\sigma_e^2=&\frac{1+\upsilon'}{3E'}\sigma_x^2+\frac{1+\upsilon}{3E}\sigma_z^2-\left(\frac{1+\upsilon'}{3E'}\cdot\frac{1+\upsilon}{3E}\right)^{1/2}\sigma_x\sigma_z+\\&+\frac{1+\upsilon'}{E'}\sigma_{xy}^2+\frac{1+\upsilon}{E}\sigma_{zx}^2\end{aligned}\,. \tag{10}$$

Hence

$$\begin{aligned}\sigma_e^2=&\frac{E(1+\upsilon')}{E'(1+\upsilon)}\sigma_x^2+\sigma_z^2-\left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2}\sigma_x\sigma_z+\\&+3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\sigma_{xy}^2+3\sigma_{zx}^2\end{aligned}\,. \tag{11}$$

The time-domain Eq. (11) is not convenient for evaluation of parameters of the reduced stress from spectral data. Therefore Eq. (11) will be transformed into the frequency domain. For this purpose the correlation theory of stochastic processes [11] will be used and Eq. (11) will be rewritten in terms of correlation functions of the processes (1) and (2) as follows

$$
\begin{aligned}
&\hat{E}\left\{\left[\overline{\sigma}_e + a_1^* \exp(-j\omega_e t_1) + a_{-1}^* \exp(j\omega_e t_1)\right]\left[\overline{\sigma}_e + a_1 \exp(j\omega_e t_2) + a_{-1} \exp(-j\omega_e t_2)\right]\right\} = \\
&= \frac{E(1+\upsilon')}{E'(1+\upsilon)} \hat{E}\left\{\left[\overline{\sigma}_x + \tilde{\sigma}_x^*(t_1)\right]\left[\overline{\sigma}_x + \tilde{\sigma}_x(t_2)\right]\right\} + \hat{E}\left\{\left[\overline{\sigma}_z + \tilde{\sigma}_z^*(t_1)\right]\left[\overline{\sigma}_z + \tilde{\sigma}_z(t_2)\right]\right\} + \\
&- \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2} \hat{E}\left\{\left[\overline{\sigma}_x + \tilde{\sigma}_x^*(t_1)\right]\left[\overline{\sigma}_z + \tilde{\sigma}_z(t_2)\right]\right\} + \\
&+ 3\frac{E(1+\upsilon')}{E'(1+\upsilon)} \hat{E}\left\{\left[\overline{\sigma}_{xy} + \tilde{\sigma}_{xy}^*(t_1)\right]\left[\overline{\sigma}_{xy} + \tilde{\sigma}_{xy}(t_2)\right]\right\} + 3\hat{E}\left\{\left[\overline{\sigma}_{zx} + \tilde{\sigma}_{zx}^*(t_1)\right]\left[\overline{\sigma}_{zx} + \tilde{\sigma}_{zx}(t_2)\right]\right\}
\end{aligned}
\tag{12}
$$

However, in analysis of fatigue safety of orthotropic materials not only the Young modulus E' and Poisson's ratio υ' but also the yield stress $R_e^{'}, R_{es}^{'}$ and fatigue limits $Z_{rc}^{'}\left(\text{or } Z_{go}^{'}\right), Z_{so}^{'}$ should be taken into account. In other words, Eq. (12) should be modified to comply with the yield stress R_e, R_{es} and fatigue limits Z_{rc}, Z_{so} as follows

$$
\begin{aligned}
&\hat{E}\left\{\left[\overline{\sigma}_e + a_1^* \exp(-j\omega_e t_1) + a_{-1}^* \exp(j\omega_e t_1)\right]\left[\overline{\sigma}_e + a_1 \exp(j\omega_e t_2) + a_{-1} \exp(-j\omega_e t_2)\right]\right\} = \\
&= \frac{E(1+\upsilon')}{E'(1+\upsilon)} \hat{E}\left\{\left[\frac{R_e}{R_e^{'}}\overline{\sigma}_x + \frac{Z_{rc}}{Z'_{rc}}\tilde{\sigma}_x^*(t_1)\right]\left[\frac{R_e}{R_e^{'}}\overline{\sigma}_x + \frac{Z_{rc}}{Z'_{rc}}\tilde{\sigma}_x(t_2)\right]\right\} + \\
&+ \hat{E}\left\{\left[\overline{\sigma}_z + \tilde{\sigma}_z^*(t_1)\right]\left[\overline{\sigma}_z + \tilde{\sigma}_z(t_2)\right]\right\} + \\
&- \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2} \hat{E}\left\{\left[\frac{R_e}{R_e^{'}}\overline{\sigma}_x + \frac{Z_{rc}}{Z'_{rc}}\tilde{\sigma}_x^*(t_1)\right]\left[\overline{\sigma}_z + \tilde{\sigma}_z(t_2)\right]\right\} + \\
&+ 3\frac{E(1+\upsilon')}{E'(1+\upsilon)} \hat{E}\left\{\left[\frac{R_{es}}{R_{es}^{'}}\overline{\sigma}_{xy} + \frac{Z_{so}}{Z'_{so}}\tilde{\sigma}_{xy}^*(t_1)\right]\left[\frac{R_{es}}{R_{es}^{'}}\overline{\sigma}_{xy} + \frac{Z_{so}}{Z'_{so}}\tilde{\sigma}_{xy}(t_2)\right]\right\} + \\
&+ 3\hat{E}\left\{\left[\overline{\sigma}_{zx} + \tilde{\sigma}_{zx}^*(t_1)\right]\left[\overline{\sigma}_{zx} + \tilde{\sigma}_{zx}(t_2)\right]\right\}
\end{aligned}
\tag{13}
$$

In accordance with Eqs (3), Eq. (13) becomes for $t_2 - t_1 = \tau$

$$\begin{aligned}
&\overline{\sigma}_e^2 + \frac{1}{4}\hat{E}\left\{a^2\right\}\left[\exp\left(j\omega_e\tau\right) + \exp\left(-j\omega_e\tau\right)\right] = \\
&= \frac{E(1+\upsilon')}{E'(1+\upsilon)}\left[\left(\frac{R_e}{R_e^{'}}\overline{\sigma}_x\right)^2 + \left(\frac{Z_{rc}}{Z_{rc}^{'}}\right)^2 K_{\sigma_x}(\tau)\right] + \overline{\sigma}_z^2 + K_{\sigma_z}(\tau) + \\
&-\left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2}\left[\frac{R_e}{R_e^{'}}\overline{\sigma}_x\overline{\sigma}_z + \frac{Z_{rc}}{Z_{rc}^{'}}K_{\sigma_x\sigma_z}(\tau)\right] + \\
&+3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left[\left(\frac{R_{es}}{R_{es}^{'}}\overline{\sigma}_{xy}\right)^2 + \left(\frac{Z_{so}}{Z'_{so}}\right)^2 K_{\sigma_{xy}}(\tau)\right] + 3\overline{\sigma}_{zx}^2 + 3K_{\sigma_{zx}}(\tau)
\end{aligned} \tag{14}$$

where:

$$\begin{aligned}
K_{\sigma_x}(\tau) &= \hat{E}\left\{\tilde{\sigma}_x^*(t_1)\tilde{\sigma}_x(t_2)\right\} \\
K_{\sigma_z}(\tau) &= \hat{E}\left\{\tilde{\sigma}_z^*(t_1)\tilde{\sigma}_z(t_2)\right\} \\
K_{\sigma_{xy}}(\tau) &= \hat{E}\left\{\tilde{\sigma}_{xy}^*(t_1)\tilde{\sigma}_{xy}(t_2)\right\} \\
K_{\sigma_{zx}}(\tau) &= \hat{E}\left\{\tilde{\sigma}_{zx}^*(t_1)\tilde{\sigma}_{zx}(t_2)\right\}
\end{aligned} \tag{15}$$

are the autocorrelation functions of the processes $\tilde{\sigma}_x, \tilde{\sigma}_z, \tilde{\sigma}_{xy}, \tilde{\sigma}_{zx}$ and

$$K_{\sigma_x\sigma_z}(\tau) = \hat{E}\left\{\tilde{\sigma}_x^*(t_1)\tilde{\sigma}_z(t_2)\right\} \tag{16}$$

is the cross correlation function of the processes $\tilde{\sigma}_x$ and $\tilde{\sigma}_z$.

Fourier transformation of Eq. (14) yields

$$\begin{aligned}
&\overline{\sigma}_e^2\delta(\omega) + \frac{1}{4}\hat{E}\left\{a^2\right\}\left[\delta(\omega-\omega_e) + \delta(\omega+\omega_e)\right] = \\
&= \left\{\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{R_e}{R_e^{'}}\overline{\sigma}_x\right)^2 + \overline{\sigma}_z^2 - \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2}\frac{R_e}{R_e^{'}}\overline{\sigma}_x\overline{\sigma}_z + 3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{R_{es}}{R_{es}^{'}}\overline{\sigma}_{xy}\right)^2 + 3\overline{\sigma}_{zx}^2\right\}\delta(\omega) + \\
&+\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{Z_{rc}}{Z_{rc}^{'}}\right)^2 S_{\sigma_x}(\omega) + S_{\sigma_z}(\omega) - \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2}\frac{Z_{rc}}{Z_{rc}^{'}}S_{\sigma_x\sigma_z}(\omega) + \\
&+3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{Z_{so}}{Z_{so}^{'}}\right)^2 S_{\sigma_{xy}}(\omega) + 3S_{\sigma_{zx}}(\omega)
\end{aligned} \tag{17}$$

where:

$S_{\sigma_x}(\omega), S_{\sigma_z}(\omega), S_{\sigma_{xy}}(\omega), S_{\sigma_{zx}}(\omega)$ — power spectral densities of the processes $\sigma_x, \sigma_z, \sigma_{xy}$ and σ_{zx};

$S_{\sigma_x \sigma_z}(\omega)$ — cross power spectral density of the processes σ_x and σ_z;

δ — Dirac's delta function.

In order to make use of the theory of energy transformation systems, Eq. (17) must be integrated over the whole frequency range [5] which gives

$$
\begin{aligned}
\overline{\sigma}_e^2 + \frac{1}{2}\hat{E}\{a^2\} = {} & \frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{R_e}{R_e'}\overline{\sigma}_x\right)^2 + \overline{\sigma}_z^2 - \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2}\frac{R_e}{R_e'}\overline{\sigma}_x\overline{\sigma}_z + \\
& + 3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{R_{es}}{R_{es}'}\overline{\sigma}_{xy}\right)^2 + 3\overline{\sigma}_{zx}^2 + \frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{Z_{rc}}{Z_{rc}'}\right)^2\int_{-\infty}^{\infty}S_{\sigma_x}(\omega)d\omega + \\
& + \int_{-\infty}^{\infty}S_{\sigma_z}(\omega)d\omega - \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2}\frac{Z_{rc}}{Z_{rc}'}\int_{-\infty}^{\infty}S_{\sigma_x\sigma_z}(\omega)d\omega + \\
& + 3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{Z_{so}}{Z_{so}'}\right)^2\int_{-\infty}^{\infty}S_{\sigma_{xy}}(\omega)d\omega + 3\int_{-\infty}^{\infty}S_{\sigma_{zx}}(\omega)d\omega
\end{aligned}
\tag{18}
$$

Hence

$$
\begin{aligned}
\overline{\sigma}_e^2 = {} & \frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{R_e}{R_e'}\overline{\sigma}_x\right)^2 + \overline{\sigma}_z^2 - \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2}\frac{R_e}{R_e'}\overline{\sigma}_x\overline{\sigma}_z + \\
& + 3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{R_{es}}{R_{es}'}\overline{\sigma}_{xy}\right)^2 + 3\overline{\sigma}_{zx}^2
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
\hat{E}\{a^2\} = 2\Bigg\{ & \frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{Z_{rc}}{Z_{rc}'}\right)^2\int_{-\infty}^{\infty}S_{\sigma_x}(\omega)d\omega + \int_{-\infty}^{\infty}S_{\sigma_z}(\omega)d\omega + \\
& - \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2}\frac{Z_{rc}}{Z_{rc}'}\int_{-\infty}^{\infty}S_{\sigma_x\sigma_z}(\omega)d\omega + 3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{Z_{so}}{Z_{so}'}\right)^2\int_{-\infty}^{\infty}S_{\sigma_{xy}}(\omega)d\omega + \\
& + 3\int_{-\infty}^{\infty}S_{\sigma_{zx}}(\omega)d\omega\Bigg\}
\end{aligned}
\tag{20}
$$

The amplitude of Gaussian process (2) follows Rayleigh distribution [11], so that

$$\hat{E}\left\{a^k\right\} = 2^{k/2}\Gamma\left(1+\frac{k}{2}\right)s_e^k\,; \quad k = 1, 2, ... \tag{21}$$

where:

s_e — standard deviation of the amplitude a;

Γ — gamma function.

In particular [9],

$$\hat{E}\{a\} = (0.5\pi)^{1/2}\,s_e\,; \tag{22}$$

$$\hat{E}\left\{a^2\right\} = 2s_e^2\,. \tag{23}$$

Equating the right-hand sides of Eqs (20) and (23) gives

$$\begin{aligned} s_e = \Bigg\{ & \frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{Z_{rc}}{Z_{rc}^{'}}\right)^2 \int_{-\infty}^{\infty} S_{\sigma_x}(\omega)d\omega + \int_{-\infty}^{\infty} S_{\sigma_z}(\omega)d\omega + \\ & -\left[\frac{E(1+\upsilon')}{E'(1+\upsilon)}\right]^{1/2} \frac{Z_{rc}}{Z_{rc}^{'}} \int_{-\infty}^{\infty} S_{\sigma_x\sigma_z}(\omega)d\omega + \\ & +3\frac{E(1+\upsilon')}{E'(1+\upsilon)}\left(\frac{Z_{so}}{Z_{so}^{'}}\right)^2 \int_{-\infty}^{\infty} S_{\sigma_{xy}}(\omega)d\omega + 3\int_{-\infty}^{\infty} S_{\sigma_{zx}}(\omega)d\omega \Bigg\}^{1/2} \end{aligned} \tag{24}$$

FATIGUE SAFETY FACTOR OF ORTHOTROPIC MATERIALS UNDER STOCHASTIC LOADS

Having determined the parameters of reduced stress at a given point of structural element made of orthotropic material and subjected to non-zero mean stochastic loads, the following formula for expected value of the fatigue safety factor can be used

$$\hat{E}\{f\} = \frac{Z_{rc}}{(0.5\pi)^{1/2}\,s_e}\left(1-\frac{\overline{\sigma}_e}{R_e}\right), \tag{25}$$

where :

s_e — the standard deviation of reduced stress amplitude defined by Eq. (24);

$\overline{\sigma}_e$ — the mean value of reduced stress calculated as

$$\overline{\sigma}_e = \left\{ \frac{E(1+\upsilon')}{E'(1+\upsilon)} \left[\left(\frac{R_e}{R_e^{'}} \overline{\sigma}_x \right)^2 + 3 \left(\frac{R_{es}}{R_{es}^{'}} \overline{\sigma}_{xy} \right)^2 \right] + \overline{\sigma}_z^2 + \right. \\ \left. + 3\overline{\sigma}_{zx}^2 - \frac{R_e}{R_e^{'}} \left[\frac{E(1+\upsilon')}{E'(1+\upsilon)} \right]^{1/2} \overline{\sigma}_x \overline{\sigma}_z \right\}^{1/2} . \tag{26}$$

Consequently, the criterion (8) of fatigue 'safe life' of orthotropic materials in the considered load case can be expressed in the form

$$\frac{1}{Z_{rc}} (0.5\pi)^{1/2} s_e + \frac{1}{R_e} \overline{\sigma}_e < 1 . \tag{27}$$

SUMMARY

In fatigue analysis, the conventional distortion energy strength theory can be utilized when the principal stress directions are constant. To avoid this requirement, in the author's papers the theory of energy transformation systems with dissipative energy as a scalar parameter has been employed which enabled the Huber-Mises-Hencky distortion energy theory to be used. The same approach is applied in the present paper to structural elements made of orthotropic materials and subjected to non-zero mean stochastic loads. As a result, the formula for expected value of fatigue safety factor in such a case is derived under assumption that the material parameters and power spectral densities of stress components at a given point of the element are known.

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ORTOTROPIA MATERIAŁU W ANALIZIE BEZPIECZEŃSTWA ZMĘCZENIOWEGO PRZY OBCIĄŻENIACH STOCHASTYCZNYCH

STRESZCZENIE

Artykuł dotyczy bezpieczeństwa zmęczeniowego elementów konstrukcyjnych wykonanych z ortotropowych metalicznych materiałów i poddanych obciążeniom stochastycznym o niezerowych wartościach średnich. Rozpatrywany jest wieloosiowy stan naprężenia przy założeniu, że znane są gęstości widmowe mocy składowych stanu naprężenia w danym punkcie elementu. Zdefiniowano naprężenie zredukowane, równoważne oryginalnemu naprężeniu w sensie wytrzymałości zmęczeniowej materiału. W tym celu wykorzystano hipotezę energii odkształcenia postaciowego i teorię systemów transformujących energię. Uwzględniono przy tym fizyczne parametry materiału odnoszące się do naprężeń działających na płaszczyźnie prostopadłej do osi ortotropowej symetrii materiału oraz parametry odnoszące się do pozostałych składowych stanu naprężenia. Wyprowadzono wzór na wartość oczekiwaną współczynnika bezpieczeństwa zmęczeniowego w takim przypadku.

Słowa kluczowe:

bezpieczeństwo zmęczeniowe, obciążenia stochastyczne, materiały ortotropowe.